DETERIORATION OF ELECTRODES SUBJECTED TO CONCENTRATED HEAT FLUXES IN A PULSE DISCHARGE

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The problem of the nonstationary thermal deterioration of a metal with the motion of the phase boundaries (melting and evaporation) taken into account is solved. The results of a computation are compared with experimental data.

Let us investigate the deterioration process within the framework of nonstationary heat conduction in a condensed substance. A thermal mechanism of deterioration holds at moderate heat-flux densities ($\lesssim 10^{10}$ W/m^2), at which the density of the material evaporates is slight and the temperature turns out to be considerably below the critical value [1, 2].

An estimate of the power of the heat source on the electrodes during a pulse discharge can be obtained from investigations of the structure of the cathode and anode domains. A continuous photographic survey of the electrode surface magnified ninefold during the pulse discharge was taken by an SFR-2M camera with 10^{-7}-sec time resolution. In the case of small interelectrode gaps (≈1 mm) it is impossible to obtain large magnifications in a frame survey, since the optic axis of the recording device should be at an angle of ≈20° to the surface of the electrode under investigation. Discharges with a Π-shaped current pulse (I = 2.5 kA) of 760 μsec duration were investigated on electrodes of different metals. The brightly glowing near-electrode spots leave tracks whose width exceeds the width of the image of the SFR-2M camera slit on the continuous photographic survey. The size of an individual spot is 0.01-0.1 mm. The length of the track left by the spots reaches a maximum value ≈30 μsec after the beginning of the discharge and corresponds to the lifetime of a fixed spot ≈10 μsec. Several tens of spots exist simultaneously on both electrodes. The process of anode and cathode domain broadening ceases ≈40 μsec after the beginning of the discharge. Up to this time a central domain of closely arranged spots of the size $l \approx 2$ mm is formed, in which the mean current density does not differ radically from the current density in an individual spot ($\lesssim 10^9$ A/m^2). Under such conditions the process of heat transport in the electrodes can be found from the following known relationships:

$$q_a = j_a(u_a + \varphi),\ q_c = \mu j_c u_c;\ \mu = 0.5 - 0.7.$$

The relationships mentioned do not reflect the complexity of the physical processes occurring in the near-electrode domains. Using them for the estimates, we obtain $q_a \leq q_c \approx 10^{10}$ W/m^2 for $j_a = j_c = 10^9$ A/m^2 for the metals W, Fe, Cu, Al, Zn, and Pb. The maximum value q_c (for tungsten) hence differs from the minimum value (for zinc) by not more than a factor of two.

Mathematically, the problem of the temperature field and the advance of the phase interface under the condition of heat transmission by conduction is described by the nonlinear heat-conduction problem with moving phase interfaces.

Let us assume that the temperature on an evaporation front is connected with the velocity of its motion by Frenkel' kinetics, the thermophysical characteristics of the medium in the solid and liquid phases are piecewise-constant, the phase transitions occur instantaneously, and the thermophysical characteristics of the medium vary by a jump during passage through the melting point T*. We neglect the change in density of the heat-conducting medium:

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial \bar{x}^2},\ \bar{y}(t) < \bar{x},\ t > 0, \tag{1}$$

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$$\frac{\partial \bar{T}}{\partial t} = \bar{a}\,\frac{\partial^2 \bar{T}}{\partial \bar{x}^2},\ \bar{y}_0(t) < \bar{x} < \bar{y}(t),\ t > 0, \tag{2}$$

$$T(\bar{y}(t),\ t) = T^*,\ \bar{T}(\bar{y}(t),\ t) = T^*,\ t \geqslant 0, \tag{3}$$

$$L\rho\,\frac{d\bar{y}}{dt} = \lambda\left(\frac{\partial T}{\partial \bar{x}}\right)\Big|_{\bar{x}=\bar{y}(t)} - \bar{\lambda}\left(\frac{\partial \bar{T}}{\partial \bar{x}}\right)\Big|_{\bar{x}=\bar{y}(t)}, \tag{4}$$

$$\bar{y}(0) = 0, \tag{5}$$

$$q + \bar{\lambda}\left(\frac{\partial \bar{T}}{\partial \bar{x}}\right)\Big|_{\bar{x}=\bar{y}_0(t)} = L_0\rho\,\frac{d\bar{y}_0}{dt} = L_0\rho v_0 \exp\left(-\frac{T_m}{T(\bar{y}_0(t),\ t)}\right), \tag{6}$$

$$T(\infty,\ t) = T_0,\ t \geqslant 0, \tag{7}$$

$$T(\bar{x},\ 0) = T_0,\ \bar{y}_0(t) \leqslant \bar{x} \leqslant \infty. \tag{8}$$

In the fixed coordinate system, coupled to the evaporation front,

$$x = \bar{x} - v_0 \int_0^t \exp\left(-\frac{T_m}{T(\bar{y}_0(t),\ t)}\right) dt$$

and the system (1)-(8) is written in dimensionless variables as

$$\frac{\partial \bar{\theta}(\xi,\ \tau)}{\partial \tau} = \frac{\partial^2 \bar{\theta}(\xi,\ \tau)}{\partial \xi^2} + u_0 \exp\left(-\frac{\theta_m}{\bar{\theta}(0,\ \tau)}\right)\cdot\frac{\partial \bar{\theta}(\xi,\ \tau)}{\partial \xi}, \tag{9}$$

$$0 < \xi < \psi(\tau),\ \tau > \tau_0,$$

$$\frac{\partial \theta(\xi,\ \tau)}{\partial \tau} = \frac{a}{\bar{a}}\,\frac{\partial^2 \theta(\xi,\ \tau)}{\partial \xi^2} + u_0 \exp\left(-\frac{\theta_m}{\theta(0,\ \tau)}\right)\cdot\frac{\partial \theta(\xi,\ \tau)}{\partial \xi},$$

$$\psi(\tau) < \xi,\ \tau > \tau_0, \tag{10}$$

$$\bar{\theta}(\psi(\tau),\ \tau) = \theta^*,\ \theta(\psi(\tau),\ \tau) = \theta^*,\ \tau \geqslant \tau_0, \tag{11}$$

$$\frac{L\rho v^0 l^0}{\bar{\lambda}T^0}\cdot\frac{d\psi(\tau)}{d\tau} = \frac{\lambda}{\bar{\lambda}}\left(\frac{\partial \theta(\xi,\ \tau)}{\partial \xi}\right)\Big|_{\xi=\psi(\tau)} - \left(\frac{\partial \theta(\xi,\ \tau)}{\partial \xi}\right)\Big|_{\xi=\psi(\tau)} - \frac{L\rho v^0 l^0}{\bar{\lambda}T^0}\,u_0 \exp\left(-\frac{\theta_m}{\bar{\theta}(0,\ \tau)}\right),\ \tau > \tau_0, \tag{12}$$

$$\psi(\tau_0) = 0, \tag{13}$$

$$\left(\frac{\partial \bar{\theta}(\xi,\ \tau)}{\partial \xi}\right)\Big|_{\xi=0} = -\frac{ql^0}{\bar{\lambda}T^0} + \frac{L_0\rho v^0 l^0}{\bar{\lambda}T^0}\,u_0 \exp\left(-\frac{\theta_m}{\bar{\theta}(0,\ \tau)}\right),\ \tau > \tau_0, \tag{14}$$

$$\theta(\infty,\ \tau) = \theta_0,\ \tau \geqslant \tau_0, \tag{15}$$

(v° and T° (the surface temperature) are taken from [3] for the stationary case). The solution of the system (10), (15), (14), and (7), written as $\theta(\xi,\ 0) = \theta_0$ for the initial stage of the pulse ($\theta(0,\ \tau) \ll \theta_m$) [4]:

$$\theta(\xi,\ \tau_0) = \frac{2ql^0}{\lambda T^0}\left\{\left(\frac{a\tau_0}{\bar{a}\pi}\right)^{1/2}\exp\left(-\frac{\bar{a}}{4a}\cdot\frac{\xi^2}{\tau_0}\right) - \frac{\xi}{2}\operatorname{erfc}\left[\frac{1}{2}\left(\frac{\bar{a}}{a\tau_0}\right)^{1/2}\xi\right]\right\} + \theta_0, \tag{16}$$

$$0 \leqslant \xi < \infty.$$

Here τ_0 is the time to achieve the melting temperature at the point $\xi = 0$.

An implicit difference scheme with phase-front capture at the nodes of the difference mesh is proposed for numerical solution of the boundary-value problem:

$$\delta_{\bar{\tau}}\bar{\theta}_{i,n} = \delta_{\xi\bar{\xi}}\bar{\theta}_{i,n} + u_0 \exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right)\delta_{\bar{\xi}}\bar{\theta}_{i,n},\quad 1 \leqslant i \leqslant n-1,\ n = 2,\ 3,\ \ldots, \tag{17}$$

$$\delta_{\bar{\tau}}\theta_{i,n} = \delta_{\xi\bar{\xi}}\theta_{i,n} + u_0 \exp\left(-\frac{\theta_m}{\theta_{0,\,n-1}}\right)\delta_{\bar{\xi}}\theta_{i,n},\quad n+1 \leqslant i \leqslant N-1,$$

$$n = 1,\ 2,\ \ldots, \tag{18}$$

$$\bar{\theta}_{n,n} = \theta^*,\ n = 0,\ 1,\ \ldots, \tag{19}$$

$$\theta_{n,n} = \theta^*,\ n = 1,\ 2,\ \ldots, \tag{20}$$

$$\frac{L\rho v^0 l^0}{\bar{\lambda}T^0}\cdot\frac{h}{\Delta\tau_n} = \frac{\lambda}{\bar{\lambda}}\delta_{\bar{\xi}}\theta_{n,n} - \delta_{\bar{\xi}}\bar{\theta}_{n-1,\,n} - \frac{L\rho v^0 l^0}{\bar{\lambda}T^0}u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right), \tag{21}$$

$$n = 1,\ 2,\ \ldots,$$

$$\delta_{\bar{\xi}}\bar{\theta}_{0,n} = -\frac{ql^0}{\bar{\lambda}T^0} + \frac{L_0\rho v^0 l^0}{\bar{\lambda}T^0}u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right),\ n = 1,\ 2,\ \ldots, \tag{22}$$

$$\theta_{N,n} = \theta_0,\ n = 0,\ 1,\ 2,\ \ldots, \tag{23}$$

$$\psi_0 = 0, \tag{24}$$

$$\theta_{i,0} = \frac{2ql^0}{\lambda T^0}\left\{\left(\frac{a\tau_0}{\bar{a}\pi}\right)^{1/2}\exp\left(-\frac{\bar{a}(ih)^2}{4a\tau_0}\right) - \frac{ih}{2}\operatorname{erfc}\left[\frac{1}{2}\left(\frac{\bar{a}}{a\tau_0}\right)^{1/2}ih\right]\right\} + \theta_0, \tag{25}$$

$$i = 0,\ 1,\ \ldots,\ N.$$

Both $\xi_i = ih$ and $\tau = \tau_n = \sum_{k=1}^{n}\Delta\tau_k$ are selected so that the ends of broken-line segment approximating the curve $\xi = \psi(\tau)$ fall at a node with the coordinates $(\psi_n = nh,\ \tau_n)$, i.e., the broken line is shifted a quantity h, the mesh spacing in ξ, along the coordinate ξ after each time spacing $\Delta\tau_n$.

We solve the nonlinear system (17)-(25) for $\theta_{i,n}$, $\bar{\theta}_{i,n}$, $\Delta\tau_n$ by iteration. Assuming that $\theta_{i,k}$, $\bar{\theta}_{i,k}$, $\Delta\tau_k$, $k = 0, 1, \ldots, n-1$ ($\Delta\tau_0$ is defined, in addition) have already been found, we determine $\theta_{i,n}$, $\bar{\theta}_{i,n}$, $\Delta\tau_n$ ($n \geq 1$) by means of the iteration formulas

$$\delta_{\bar{\tau}}\theta^{(s)}_{i,n} = \frac{a}{\bar{a}}\delta_{\xi\bar{\xi}}\theta^{(s)}_{i,n} + u_0\exp\left(-\frac{\theta_m}{\theta_{0,\,n-1}}\right)\delta_{\bar{\xi}}\theta^{(s)}_{i,n},\ n+1 \leqslant i \leqslant N-1, \tag{26}$$

$$\delta_{\bar{\tau}}\theta^{(s)}_{i,n} = \frac{1}{\Delta\tau^{(s)}_n}(\theta^{(s)}_{i,n} - \theta_{i,\,n-1}), \tag{27}$$

$$\theta^{(s)}_{n,n} = \theta^*,$$

$$\theta^{(s)}_{N,n} = \theta_0, \tag{28}$$

$$\delta_{\bar{\tau}}\bar{\theta}^{(s)}_{i,n} = \delta_{\xi\bar{\xi}}\bar{\theta}^{(s)}_{i,n} + u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right)\delta_{\bar{\xi}}\bar{\theta}^{(s)}_{i,n},\ 1 \leqslant i \leqslant n-1, \tag{29}$$

$$\delta_{\bar{\xi}}\bar{\theta}^{(s)}_{0,n} = -\frac{ql^0}{\bar{\lambda}T^0} + \frac{L_0\rho v^0 l^0}{\bar{\lambda}T^0}u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right), \tag{30}$$

$$\bar{\theta}^{(s)}_{n,n} = \theta^*,$$

$$\Delta\tau^{(s+1)}_n = \left[\frac{L\rho v^0 l^0}{\bar{\lambda}T^0}h + \Delta\tau^{(s)}_n\left(\frac{ql^0}{\bar{\lambda}T^0} - \frac{L_0\rho v^0 l^0}{\bar{\lambda}T^0}u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right) + \right.\right. \tag{31}$$

$$\left.\left. + \delta_{\bar{\xi}}\bar{\theta}^{(s)}_{n-1,\,n} - \frac{\lambda}{\bar{\lambda}}\delta_{\bar{\xi}}\theta^{(s)}_{n,n} + \frac{\lambda}{\bar{\lambda}}\delta_{\bar{\xi}}\theta^{(s)}_{N-1,\,n}\right)\right]\left(\frac{ql^0}{\bar{\lambda}T^0} - \frac{(L_0+L)\rho v^0 l^0}{\bar{\lambda}T^0}u_0\exp\left(-\frac{\theta_m}{\bar{\theta}_{0,\,n-1}}\right) + \frac{\lambda}{\bar{\lambda}}\delta_{\bar{\xi}}\theta^{(s)}_{N-1,\,n}\right)^{-1},$$

$$\Delta\tau_1^{(0)} = \frac{L\rho v^0 l^0}{\bar{\lambda}T^0}\, h\left(\frac{ql^0}{\bar{\lambda}T^0} - \frac{(L_0 + L)\,\rho v^0 l^0}{\bar{\lambda}T^0}\, u_0 \exp\left(-\frac{\theta_m}{\bar{\theta}_{0,0}}\right) + \frac{\lambda}{\bar{\lambda}}\,\delta_{\xi}\theta_{N-1,\,0}\right)^{-1},$$

$$\Delta\tau_n^{(0)} = \Delta\tau_{n-1},\ (n \geqslant 2), \tag{32}$$

where s = 0, 1, 2, ... is the number of the iteration.

If $\Delta\tau_n^{(s)} > 0$ is known, then by solving the two linear algebraic systems (26)-(28) and (29)-(31) with the initial condition (25) for s, we obtain $\theta_{i,n}^{(s)}$, $\bar{\theta}_{i,n}^{(s)}$ and $\Delta\tau_n^{(s+1)}$ from (32). The mentioned systems of equations were solved by the factorization method [5] in each iteration. Iteration was continued until agreement between two successive iterations with a given ε-accuracy

$$\max_i \frac{|\bar{\theta}_{i,n}^{(s+1)} - \bar{\theta}_{i,n}^{(s)}|}{\bar{\theta}_{i,n}^{(s)}} < \varepsilon,\ \max_i \frac{|\theta_{i,n}^{(s+1)} - \theta_{i,n}^{(s)}|}{\theta_{i,n}^{(s)}} < \varepsilon,$$

$$\frac{|\Delta\tau_n^{(s+1)} - \Delta\tau_n^{(s)}|}{\Delta\tau_n^{(s)}} < \varepsilon,$$

after which we set

$$\bar{\theta}_{i,n} = \bar{\theta}_{i,n}^{(s+1)},\ \theta_{i,n} = \theta_{i,n}^{(s+1)},\ \Delta\tau_n = \Delta\tau_n^{(s+1)}.$$

It can be shown by methods elucidated in [6, 7] that the solution of the boundary-value problem (9)-(16) exists and is unique and convergence of the approximate solution of the difference problem (26)-(32) with (25) to the solution of the differential problem (9)-(16) occurs.

Taking account of the temperature dependences of the thermophysical parameters of the problem (including L_0), as well as extension to the case of an arbitrarily shaped pulse, are not difficult in the method (26)-(32), in principle.

A number of specific versions of the problem for the metals presented in Table 1 was checked by the method elucidated for a flux $q = 10^{10}$ W/m².

The results of computing the surface temperature T_{surf}, the melting u_{melt}, and evaporation u_{eva} rates, and the melting and evaporation front coordinates as a function of the time are presented in Table 2. For all the versions N = 1200 was taken in the computation, while the times to reach the melting point on the surface, the magnitudes of the spacings in the space variable, and the ratio between the evaporated mass and the mass of the melt up to the end of the pulse equalled, respectively, $t_0 = 3.464 \cdot 10^{-5}$ sec, $h = 3.125 \cdot 10^{-3}$, 15.82% for

TABLE 1. Thermophysical Characteristics of the Metals Investigated

Metal	W	Fe	Cu	Al	Zn	Pb
$\bar{\lambda}$, W/m·deg	80	36,4	315	156	70	18
λ, W/m·deg	125	54,6	377	251	110	34
$\bar{a}\cdot 10^5$, m²/sec	2,6	0,8	9,3	7	1,9	1,1
$a\cdot 10^5$, m²/sec	4	1,2	11,1	11	4,16	2,29
$L_0\rho\cdot 10^{-10}$, W·sec/m³	8,92	5,63	4,9	3,18	1,43	0,959
$L\rho\cdot 10^{-9}$, W·sec/m³	3,72	2,2	1,9	1,0	0,82	0,323
v_0, m/sec	5174	4910	3666	5100	3700	1260
T^*, °K	3653	1812	1356	932	692,7	600
T_m, °K	101000	48500	40200	38000	15600	24300
T_0, °K	293	293	293	293	293	293

TABLE 2. Results of Computing Electrode Deterioration

	$t \cdot 10^6$, sec	T_{surf}, °K	v_{mp}, $\frac{m}{sec}$	u_{eva}, $\frac{m}{sec}$	$\bar{y}(t) \cdot 10^5$, m	$\bar{y}_0(t) \cdot 10^5$, m
W	55,94	3773	0,04525	$1,229 \cdot 10^{-8}$	$9,638 \cdot 10^{-2}$	$2,618 \cdot 10^{-8}$
	77,82	4858	0,4159	$4,829 \cdot 10^{-6}$	0,9638	$2,645 \cdot 10^{-6}$
	100,3	6046	0,4365	$2,880 \cdot 10^{-4}$	1,928	$2,106 \cdot 10^{-4}$
	122,3	7110	0,4408	$3,502 \cdot 10^{-3}$	2,895	$3,717 \cdot 10^{-3}$
	144,6	7827	0,4390	$1,286 \cdot 10^{-2}$	3,877	$2,217 \cdot 10^{-2}$
	192,5	8460	0,4149	$3,381 \cdot 10^{-2}$	5,922	0,1395
	247,1	8718	0,3764	$4,812 \cdot 10^{-2}$	8,079	0,3689
	346,7	8905	0,3199	$6,142 \cdot 10^{-2}$	11,53	0,9253
	476,4	9003	0,2643	$6,949 \cdot 10^{-2}$	15,28	1,782
	588,0	9047	0,2276	$7,331 \cdot 10^{-2}$	18,00	2,582
	654,1	9053	0,2250	$7,388 \cdot 10^{-2}$	19,47	3,081
Fe	7,222	1894	0,1103	$3,736 \cdot 10^{-8}$	$2,999 \cdot 10^{-2}$	$1,016 \cdot 10^{-8}$
	10,13	2635	0,9730	$4,972 \cdot 10^{-5}$	0,2999	$3,301 \cdot 10^{-6}$
	13,15	3406	0,9991	$3,207 \cdot 10^{-3}$	0,6002	$3,278 \cdot 10^{-4}$
	16,21	3924	0,9789	$2,106 \cdot 10^{-2}$	0,9038	$3,965 \cdot 10^{-3}$
	23,06	4300	0,8601	$6,200 \cdot 10^{-2}$	1,534	$3,455 \cdot 10^{-2}$
	31,55	4429	0,7167	$8,619 \cdot 10^{-2}$	2,199	$9,929 \cdot 10^{-2}$
	42,39	4494	0,5898	0,1010	2,902	0,2022
	56,74	4533	0,4729	0,1108	3,655	0,3553
	76,79	4568	0,3902	0,1202	4,485	0,5861
Cu	17,84	1400	0,2152	$1,247 \cdot 10^{-9}$	0,1394	$8,076 \cdot 10^{-10}$
	32,05	1798	0,9192	$7,191 \cdot 10^{-7}$	1,394	$2,448 \cdot 10^{-7}$
	46,85	2239	0,9564	$5,836 \cdot 10^{-5}$	2,788	$2,651 \cdot 10^{-5}$
	75,73	3043	0,9722	$6,703 \cdot 10^{-3}$	5,581	$5,743 \cdot 10^{-3}$
	105,2	3482	0,9568	$3,545 \cdot 10^{-2}$	8,431	$6,739 \cdot 10^{-2}$
	136,7	3667	0,9208	$6,356 \cdot 10^{-2}$	11,38	0,2276
	170,7	3763	0,8676	$8,396 \cdot 10^{-2}$	14,42	0,4830
	208,2	3821	0,8105	$9,889 \cdot 10^{-2}$	17,56	0,8285
	271,5	3876	0,7277	0,1147	22,42	1,511
	345,4	3911	0,6497	0,1258	27,49	2,405
	432,9	3934	0,5724	0,1339	32,82	3,545
	580,2	3956	0,4687	0,1417	40,43	5,583
	762,0	3974	0,3740	0,1483	48,08	8,219
Al	3,814	982,4	0,3982	$8,102 \cdot 10^{-14}$	$7,863 \cdot 10^{-2}$	0
	9,218	1436	1,354	$1,637 \cdot 10^{-8}$	0,7863	$1,507 \cdot 10^{-9}$
	14,92	1937	1,394	$1,540 \cdot 10^{-5}$	1,473	$2,040 \cdot 10^{-6}$
	20,55	2423	1,397	$7,895 \cdot 10^{-4}$	2,359	$1,509 \cdot 10^{-4}$
	31,91	3155	1,396	$2,996 \cdot 10^{-2}$	3,945	$1,352 \cdot 10^{-2}$
	43,80	3436	1,359	$8,019 \cdot 10^{-2}$	5,586	$8,166 \cdot 10^{-2}$
	56,64	3560	1,298	0,1178	7,288	0,2117
	78,06	3653	1,187	0,1549	9,945	0,5091
	102,6	3704	1,077	0,1787	12,72	0,9228
	131,0	3736	0,9707	0,1951	15,61	1,456
	164,1	3758	0,8676	0,2070	18,64	2,124
	206,2	3775	0,7605	0,2166	22,05	3,017
Zn	0,7368	726,3	0,6318	$1,736 \cdot 10^{-6}$	$2,349 \cdot 10^{-2}$	$6,455 \cdot 10^{-8}$
	1,433	1027	3,179	$9,396 \cdot 10^{-4}$	0,2349	$1,652 \cdot 10^{-5}$
	2,166	1325	3,223	$2,861 \cdot 10^{-2}$	0,4707	$8,393 \cdot 10^{-4}$
	2,920	1502	3,144	0,1144	0,7111	$6,395 \cdot 10^{-3}$
	4,635	1634	2,746	0,2649	1,216	$4,114 \cdot 10^{-2}$
	6,764	1684	2,334	0,3515	1,753	0,1084
	9,491	1710	1,924	0,4037	2,327	0,2126
	13,12	1725	1,559	0,4381	2,950	0,3661
	18,23	1736	1,223	0,4621	3,651	0,5976
	21,74	1745	1,134	0,4848	4,052	0,7627
Pb	0,07219	643,0	2,225	$1,980 \cdot 10^{-13}$	$7,747 \cdot 10^{-3}$	$6,894 \cdot 10^{-16}$
	0,1313	1027	12,30	$1,617 \cdot 10^{-7}$	$7,747 \cdot 10^{-2}$	$1,601 \cdot 10^{-10}$
	0,1948	1422	11,98	$9,031 \cdot 10^{-5}$	0,1549	$1,492 \cdot 10^{-7}$
	0,2614	1789	11,40	$2,620 \cdot 10^{-3}$	0,2324	$6,733 \cdot 10^{-6}$
	0,4036	2386	10,55	$6,950 \cdot 10^{-2}$	0,3878	$4,057 \cdot 10^{-4}$
	0,7307	2831	8,716	0,3237	0,7046	$7,372 \cdot 10^{-3}$
	1,423	2998	5,974	0,5142	1,200	$3,788 \cdot 10^{-2}$
	2,517	3060	3,942	0,6018	1,727	$9,994 \cdot 10^{-2}$
	4,735	3090	2,386	0,6486	2,331	0,2395
	9,363	3106	1,176	0,6731	3,026	0,5467

TABLE 3. Results of an Experimental Investigation of Electrode Deterioration

Electrode material	$\tilde{\tau}\cdot10^6$, sec	$U\cdot10^{-3}$, V	$\bar{x}_c\cdot10^5$, m	$\bar{x}_a\cdot10^5$, m
W	205	0,8	0,77	3,1
	350	1,75	2,3	6,3
	590	1,75	3,4	15
Fe	55	0,85	0,63	1,9
Cu	205	0,8	8,5	8,5
	350	1,75	13	11
	590	1,75	20	22
Al	55	0,85	1,8	7,4
	205	0,8	9,3	19
Zn	18	1,7	2,0	3,4
Pb	9	1,8	6,7	8,7

tungsten; $t_0 = 4.502\cdot10^{-6}$, $h = 6.25\cdot10^{-3}$, 13.07% for iron; $t_0 = 1.136\cdot10^{-5}$ sec, $h = 2.5\cdot10^{-3}$, 17.09% for copper; $t_0 = 1.840\cdot10^{-6}$ sec, $h = 3.125\cdot10^{3}$, 13.69% for aluminum; $t_0 = 3.737\cdot10^{-8}$ sec, $h = 6.25\cdot10^{-3}$, 18.07% for lead; and $t_0 = 3.650\cdot10^{-7}$ sec, $h = 6.25\cdot10^{-3}$, 18.83% for zinc. The error in the computation does not exceed 0.5% of the values presented in Table 2 for copper and aluminum and 1% for the rest of the metals.

The nonstationarity of the process during a significant part of the duration of the pulses under investigation should be noted. The temperature at the surface from the time of the beginning of the melting increased severalfold, hence exceeding the provisional boiling point which assures effective elimination of vapor from the metal surface. In the cases analyzed the size of the melt zone is a maximum to the end of the heat pulse and agrees in order of magnitude with the maximum size of a liquid-metal drop ejected from the electrodes during the pulse discharges. It is known that it is impossible to make such a deduction under the assumption of higher current densities on the electrodes [8]. This circumstance is a graphic qualitative foundation for the applicability of the model presented above for analysis of the melt zone.

The deterioration of electrodes with a bounded working surface size was investigated experimentally. The discharges were carried out between two flat end faces of cylindrical electrodes. Reliable insulation of the side surface of the electrodes was assured by compact fitting of the polyvinyl tube. The mutually perpendicular arrangement of the electrodes eliminated torch deterioration. The diameter of the electrode under investigation was selected to equal 1.35 mm for lead and zinc and 1.6 mm for the other metals. The rectangular current pulses for which the duration of the flat top $\tilde{\tau}$ is indicated in Table 3, were shaped by artificial LC-lines with from 5-18 cells. The capacitor charging voltage U was selected so that the current density on the electrodes would be 10^9 A/m^2 for all the discharges. The magnitude of electrode erosion was determined by weighings on analytical balances before and after a unit discharge. For convenience of a comparison with the results in Table 2, the coordinate of the boundary of electrode deterioration after the discharge was computed in terms of the mass of eroded metal, the electrode diameter, and its density. The slight curvature of the electrode surface after the discharge makes such a conversion physically correct. The results are represented in Table 3. The root-mean-square deviation from the values presented in the table does not exceed 30% (safety factor 0.98) for nine measurements of the magnitude of the electrode erosion).

As is seen from Tables 2 and 3, the coordinate of the deterioration boundary (the experimental value) is near the evaporation boundary for the refractory metals and near the melting boundary for the easily melted metals. In the latter case, the ejection of a large quantity of the liquid phase is observed. A detailed analysis of the correspondence between the computed and experimental values is possible only in the presence of reliable methods of determining the phase composition of the products of the electrical erosion.

NOTATION

q_a, q_c, heat flux density on the anode and cathode; u_a, u_c, anode and cathode voltage drops; φ, output potential; μ, coefficient taking into account the fraction of ions in the

current on the surface, and the ion accommodation and neutralization processes; T, $\bar{T}$, a, $\bar{a}$, λ, $\bar{\lambda}$, temperature, thermal diffusivity, and thermal conductivity in the solid and liquid phases, respectively; T_m, characteristic temperature; ρ, density; L, L_0, latent heats of melting and evaporation; v_0, maximum velocity of evaporation-front motion; x, running coordinate; t, time; $y_0(t)$, $y(t)$, coordinates of the evaporation and melting fronts; $\tau = t/t^\circ$, $\xi = x/l^\circ$, $\theta = T/T^\circ$, $\psi(t) = y(t)/l^\circ$, $u_0 = v_0/v^\circ$, dimensionless time, running coordinate, temperature, melting-front coordinate, and maximum velocity of evaporation front motion; $t^\circ = \bar{a}/(v^\circ)^2$, $l^\circ = v^\circ t^\circ$, T°, characteristic time, length, and temperature; v°, evaporation-wave velocity in the body:

$$\delta_\xi \theta_{i,n} = \frac{1}{h}(\theta_{i+1,n} - \theta_{i,n}), \quad \delta_{\xi\xi} \theta_{i,n} = \frac{1}{h^2}(\theta_{i+1,n} - 2\theta_{i,n} + \theta_{i-1,n}),$$

$$\delta_\tau \theta_{i,n} = \frac{1}{\Delta\tau_n}(\theta_{i,n} - \theta_{i,n-1});$$

$\tilde{\tau}$, duration of the flat top of the current pulse; U, charging voltage on the capacitors; $\bar{x}_a$, coordinate of the deterioration boundary (experiment) on the anode; $\bar{x}_c$, coordinate of the deterioration boundary (experiment) on the cathode.

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CHOOSING A CALCULATION SCHEME FOR THE TEMPERATURE FIELD IN PULSE MICROWELDING

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Thermal processes in pulsed methods of microwelding in a solid phase during semiconductor assembly are investigated. Expressions determining the choice of the temperature-field calculational scheme are subjected to numerical analysis. It is shown that the calculated and experimental data are in qualitative agreement.

In choosing the optimal technology and equipment for the assembly of microelectronic devices, an important stage is the determination of the acceptable thermal effect on semiconductor elements of the microwelding in the solid phase of a small-diameter wire to contact areas on its surface.

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